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by

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BALLISTIC RESEARCH LABORATORIES
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R. W. Deas
W. M. Werner

Interior Ballistics Laboratory

RDT&E Project No. 1T013001A91A

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B A L L I S T I C   R E S E A R C H   L A B O R A T O R I E S

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ABSTRACT

The theory of single-wire torsional pendulums, including second-order effects, is discussed. The theory is applied to the use of torsional pendulums in measuring moments of inertia. Two types of torsional pendulums designed for moment of inertia measurements are described; one for odd-shaped bodies and one for artillery projectiles. A data reduction program for digital computers is included.

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LIST OF SYMBOLS

d	diameter of suspension wire, ft
e	magnitude of static unbalance, ft
k	coefficient of restoring torque, ft-lb
ℓ	length of the pendulum suspension wire, ft
m	mass, lb-sec^2 per ft
n	positive integer
r	distance between a suspension strand and the axis of rotation in case of multi-strand suspension, ft
C	damping coefficient, ft-lb-sec
E	Young's modulus, lb per ft^2
G	modulus of rigidity, lb per ft^2
I	moment of inertia, ft-lb-sec^2
L	length of optical path from pendulum to photocell, ft
R	distance between the center of gravity of an object and an axis, ft
T	period of torsional oscillation, sec
V	speed of light beam traversing photocell, ft per sec
W	weight, lb
β	coefficient of linear thermal expansion, per $^{\circ}$C
γ	Poisson's ratio, dimensionless
θ	angular deflection, degrees
ρ	radius of axial holder, ft
τ	temperature, $^{\circ}$C
ω	angular frequency, degrees per sec
Θ	amplitude of oscillation, degrees

LIST OF SYMBOLS (Continued)

Subscripts:

a	axial
b	pendulum bob
h	holder
i	i^{th} calibration mass
o	standard conditions
t	transverse

Superscripts:

—	average condition
.	first derivative with respect to time
..	second derivative with respect to time

1. INTRODUCTION

To relate the observed motion of a rigid body with the motion predicted by theoretical analysis, it is necessary to determine the dynamic specifications of the body. In particular, when dealing with a rotating body, the moments of inertia of the body about the axes of rotation must be known. Where the body is simply shaped and of uniform density, it is not difficult to calculate its moments of inertia. However, most real systems involve bodies having complex configurations and parts of different density. For instance, the calculation of the moments of inertia for a rifle is impractical. Therefore, some sort of measuring instrument is desirable. This report is a general study of the single-wire torsional pendulum as a device for measuring moments of inertia, and includes a description of two examples and methods for their use.

2. ANALYSIS OF THE TORSIONAL PENDULUM

2.1 General Remarks on Torsional Pendulums

The rotational motion, θ, of a rigid body about an axis depends on the moment of inertia, I_b, of the body about the axis of rotation. The applicable differential equation for undamped, free, small oscillations is

$$I_b \ddot{\theta} + k\theta = 0 \quad , \tag{1}$$

where k = coefficient of restoring torque, ft-lb.

The resulting frequency equation provides a relation between the moment of inertia and the period of the torsional oscillation, T, which is

$$I_b = \frac{k}{4\pi^2} T^2 \quad . \tag{2}$$

This relation is utilized in torsional pendulum techniques of determining the moment of inertia of bodies. The body to be measured is attached to a suspension consisting of one or more vertical strands, usually wires with fixed ends. The body is oriented so that the axis of rotation passes through the center of gravity of the body. The period of

oscillation about the axis of rotation is measured, and the moment of inertia is determined from Equation (2). The constant of proportionality $\frac{k}{4\pi^2}$ depends on the method of suspension. For a single wire with a circular cross section, strength of material considerations lead to

$$k = \frac{\pi G d^4}{32 \ell} , \tag{3}$$

where G = modulus of rigidity, lb per ft^2,

d = diameter of wire, ft,

ℓ = length of wire, ft.

For a suspension made up of several such wires equally spaced from the axis of rotation,

$$k = \frac{W_b r^2}{\ell} + \frac{n \pi G d^4}{32 \ell} , \tag{4}$$

where W_b = weight of pendulum bob, lb,

r = distance of wires from axis of rotation, ft,

n = number of wires.

Higher order terms are avoided by restricting the motion to small amplitude.

This relation, Equation 4, affords considerable latitude to the designer of hardware in the choice of values for the variables. A multi-wire suspension permits reducing to negligible the dependence on the modulus of rigidity by increasing r and decreasing d. Such a pendulum is suitable for relatively massive objects for which a single wire is impractical. However, since k is dependent on weight, a separate determination of k is required for each end load. A graph of I_b versus T^2 is then a family of straight lines with the end load as parameter.

The quantity k for a single-wire pendulum is unaffected by weight except for second-order effects. In this case $r = 0$ and $n = 1$ in Equation (4) and a graph of I_b versus T^2 consists of a single straight line. Since a single strand allows the pendulum to tip, balancing can be accomplished with additional hardware. The period of oscillation can be made suitable for available timing devices by the proper choice of the diameter and length of the suspension wire.

From the standpoint of simplicity of hardware and data handling a single-wire suspension with fixed ends was selected for the pendulums described in this report. One configuration provided both the means of locating the center of gravity and for measuring the moment of inertia of an odd-shaped body. Another configuration enables the precise determination of the moments of inertia of artillery projectiles.

2.2 First-Order Theory of the Single-Wire Torsional Pendulum

If as is the usual practice a holding device is used to support the object to be measured, the moment of inertia of the pendulum bob, I_b, will be the sum of the moment of inertia of the holder, I_h, and the moment of inertia of the object, I, each taken about the common axis of rotation.

$$I_b = I + I_h \tag{5}$$

I_h will be constant if the holder always oscillates about the same axis. In practice this is accomplished by balancing the holder alone so that its center of gravity is colinear with the suspension wire. Then, an object is placed in the holder and shifted until the holder regains its balanced position, which implies that the center of gravity of the object, as well as that of the holder, is colinear with the suspension wire. If this balancing procedure is followed for all objects, then the holder will always have a constant orientation with respect to the vertical suspension wire, and I_h will be constant.

Using Equations (2) and (3) we obtain *

$$I = \frac{k}{4\pi^2} T^2 - I_h \ . \tag{6}$$

The validity of Equation (6) might be questioned because of the absence of damping. However, in practice air damping has a negligible effect on the determination of I as illustrated in Appendix I.

The apparatus is calibrated by determining the constants k and I_h. This is best accomplished by timing the periods, T_i, of the pendulum when loaded with calibration masses whose moments of inertia I_i are known. Substitution of the measured periods into Equation (6) gives a system of simultaneous equations,

$$I_i = \frac{k}{4\pi^2} T_i^{\,2} - I_h, \qquad i = 1, 2, \ldots n, \qquad (7)$$

which is solved for k and I_h. For $n > 2$, the number of equations exceeds the number of unknowns, and a least squares solution for k and I_h is possible. Such a procedure minimizes the inaccuracy resulting from random errors in the measurement of the periods.

Typical calibration masses are shown in Figure 1. Right circular cylinders or rectangular prisms are used because of the ease in machining and the simplicity in calculating their moments of inertia.

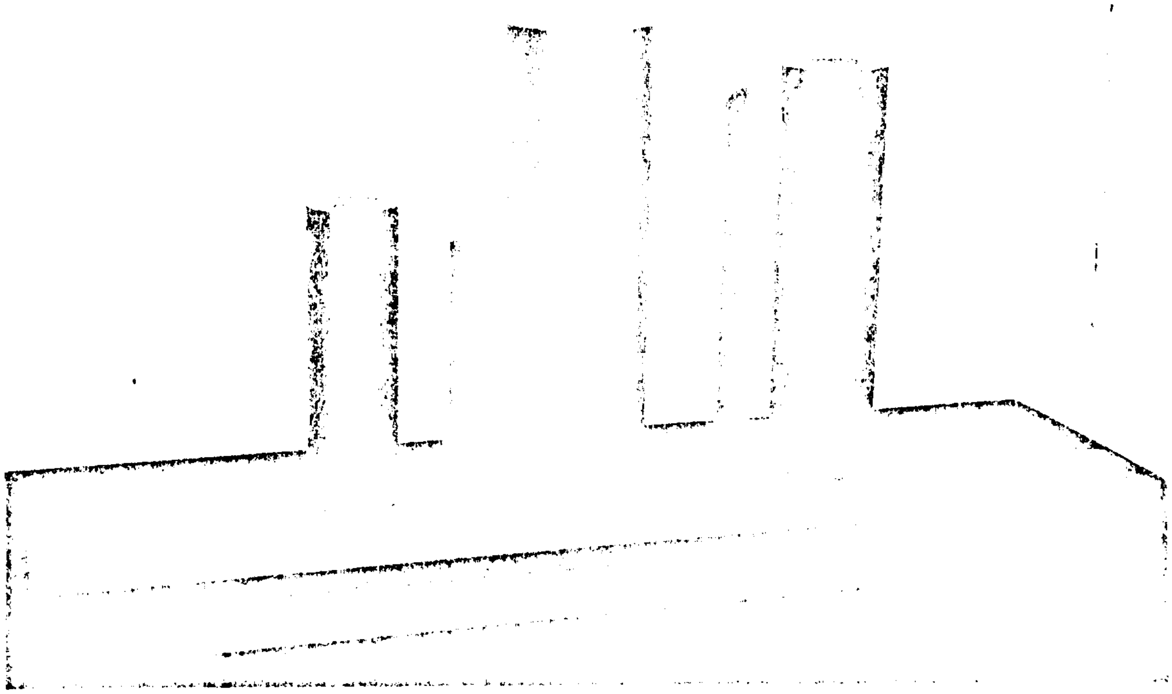

Figure 1. Typical calibration masses for the torsional pendulum

After calibration the body to be measured is placed in the holder and aligned to make all centers of gravity colinear with the suspension wire. The pendulum is set into torsional oscillation about the suspension and the period, T, is measured. The moment of inertia is then calculated using Equation (6).

2.3 Corrections for Variations in Temperature, End Load, and Amplitude of Oscillation

The method outlined in 2.2 is based on the assumption that k, I_h, and I_i are constants. This approximation is good enough to give results with accuracy on the order of 1 percent. However, for very accurate measurements the effects of variables such as end load, temperature, and amplitude of oscillation must be taken into account. It is usually impractical to hold these quantities constant for any extended period. Therefore, if the functional dependence of k, I_h and I_i upon the variables is unknown, frequent calibration is necessary for very accurate results. An alternate method, which eliminates the need for frequent calibrations and at the same time yields highly accurate measurements, involves the use of corrections for the disturbing factors. The corrections for the effects of temperature, end load, and amplitude of oscillation are determined in the following manner:

(1) A set of standard conditions of temperature, end load and amplitude of oscillation is selected. The restoring torque of the wire and the moments of inertia of the holder and the i^{th} calibration mass under these standard conditions are denoted by k_o, I_{ho} and I_{io}, respectively.

(2) Expressions for k, I_h and I_i as functions of k_o, I_{ho}, I_{io} and the deviations from the standard conditions are developed.

(3) The expressions for k, I_h and I_i are substituted into the set of calibration Equations (7), which is then solved for k_o and I_{ho}.

(4) The values of k_o and I_{ho} are substituted into Equation (6) to yield the unknown moment of inertia in terms of T, k_o, I_{ho} and the observed deviations from standard conditions at the time of measurement.

Assuming the following standard conditions, temperature τ_o, end load W_o, and amplitude of oscillation Θ_o, the restoring torque k at other than standard conditions is given by

$$k = k_o + dk_o \approx k_o + \frac{\partial k}{\partial \tau}(\tau - \tau_o) + \frac{\partial k}{\partial W}(W - W_o) + \frac{\partial k}{\partial \Theta}(\Theta - \Theta_o)$$

$$= k_o \left[1 + \frac{1}{k_o}\frac{\partial k}{\partial \tau}(\tau - \tau_o) + \frac{1}{k_o}\frac{\partial k}{\partial W}(W - W_o) + \frac{1}{k_o}\frac{\partial k}{\partial \Theta}(\Theta - \Theta_o)\right]. \quad (8)$$

The moments of inertia of the holding device and the calibration masses are functions of temperature. The moment of inertia of the holder is given by

$$I_h = \int R^2 dm ,$$

where R = distance between the center of gravity of an element of mass dm and the axis of rotation. R, however, is a function of temperature:

$$R = R_o \left[1 + \beta_h (\tau - \tau_o)\right],$$

where β_h = coefficient of thermal expansion of the holder, per ^oC,
R_o = distance, ft, between the element of mass dm and the axis of rotation at temperature τ_o.

Thus,

$$I_h = \left[1 + \beta_h (\tau - \tau_o)\right]^2 \int R_o^{\,2}\, dm$$

$$= \left[1 + \beta_h (\tau - \tau_o)\right]^2 I_{ho} .$$

Or,

$$I_h \approx \left[1 + 2\beta_h (\tau - \tau_o)\right] I_{ho}. \quad (9)$$

Using the same reasoning, it follows that

$$I_i \approx \left[1 + 2\beta_i (\tau_i - \tau_o)\right] I_{io} , \quad (10)$$

where β_i = coefficient of thermal expansion of i^{th} calibration mass, per ^{o}C,

τ_i = temperature of i^{th} calibration mass.

Equations (8), (9) and (10) can then be used to express Equation (7) as follows,

$$I_{io}\,[1 + \beta_i\,(\tau_i - \tau_o)] =$$

$$= \frac{k_o}{4\pi^2}\left[1 + \frac{1}{k_o}\frac{\partial k}{\partial \tau}(\tau_i - \tau_o) + \frac{1}{k_o}\frac{\partial k}{\partial W}(W_i - W_o) + \frac{1}{k_o}\frac{\partial k}{\partial \Theta}(\Theta_i - \Theta_o)\right] T_i^{\,2}$$

$$- I_{ho}\,[1 + 2\beta_h\,(\tau_i - \tau_o)] \qquad i = 1, 2, \ldots n. \tag{11}$$

Likewise, Equation (6) relating period and moment of inertia can be expressed as

$$I = \frac{k_o}{4\pi^2}\left[1 + \frac{1}{k_o}\frac{\partial k}{\partial \tau}(\tau - \tau_o) + \frac{1}{k_o}\frac{\partial k}{\partial W}(W - W_o) + \frac{1}{k_o}\frac{\partial k}{\partial \Theta}(\Theta - \Theta_o)\right] T^2 - I_{ho}\,[1 + 2\beta_h\,(\tau - \tau_o)]. \tag{12}$$

Ideally, the coefficients $\frac{1}{k_o}\frac{\partial k}{\partial \tau}$, $\frac{1}{k_o}\frac{\partial k}{\partial W}$, and $\frac{1}{k_o}\frac{\partial k}{\partial \Theta}$ could be evaluated from theoretical considerations leading from Equation (3),

$$k = \frac{\pi G d^4}{32\ell}. \tag{3}$$

For example, forming the thermal coefficient of k leads to

$$\frac{1}{k_o}\frac{\partial k}{\partial \tau} = \frac{1}{G_o}\frac{\partial G}{\partial \tau} + \frac{4}{d}\frac{\partial d}{\partial \tau} - \frac{1}{\ell}\frac{\partial \ell}{\partial \tau}$$

$$= \frac{1}{G_o}\frac{\partial G}{\partial \tau} + 3\beta, \tag{13}$$

where β is the coefficient of linear thermal expansion of the suspension wire, per ^{o}C.

While values for β are usually available, such coefficients as $\frac{1}{G}\frac{\partial G}{\partial \tau}$ are not. Therefore, it is necessary to determine the coefficients experimentally. The procedure for doing this is as follows:

(1) k_o is determined under the standard conditions.

(2) k is measured while one parameter is allowed to vary and the others are kept constant.

(3) k/k_o is determined as a function of the varied parameter. The derivative of k/k_o with respect to the varied parameter is the desired coefficient.

As an example of the above procedure, plots of k/k_o versus τ, W and Θ for a 1/16 inch diameter steel wire are given in Figures 2, 3 and 4. The slopes of the curves yield the coefficients $\frac{1}{k_o}\frac{\partial k}{\partial \tau}$, $\frac{1}{k_o}\frac{\partial k}{\partial W}$ and $\frac{1}{k_o}\frac{\partial k}{\partial \Theta}$, thus enabling the precise calculation of k for conditions other than standard. A complicated correction to k is required when a magnetized body is being measured. It is usually simpler and better to demagnetize the body before measurement.

The hand reduction of the calibration and measurement data is tedious when Equations (11) and (12) are used. Because of this, it is advantageous to use computers in the data reduction. A program, written for the BRLESC[1,2*] computer to obtain solutions for k_o and I_{ho} using the method of least squares is included in Appendix B. This program further computes moments of inertia using Equation (12) and the computed values of k_o and I_{ho} for any number of measurements. Typical input data and results are also given.

*Superscript numbers denote references which may be found on page 33.

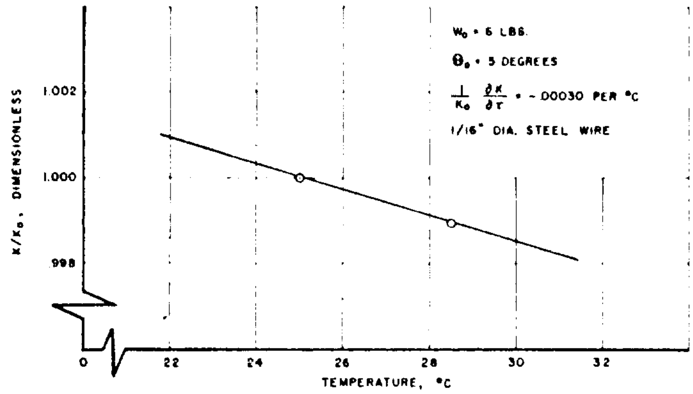

Figure 2. $\frac{K}{K_o}$ vs. temperature with fixed end load and amplitude of oscillation

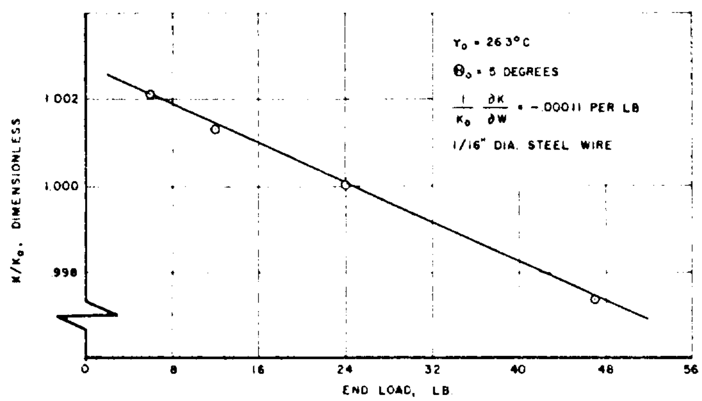

Figure 3. $\frac{K}{K_o}$ vs. end load at fixed temperature and amplitude of oscillation

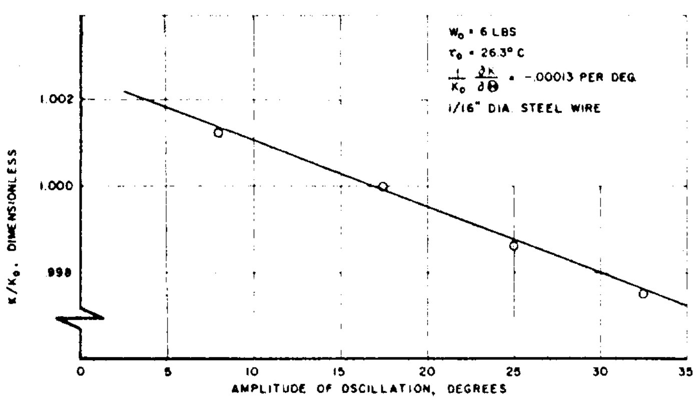

Figure 4. $\frac{K}{K_o}$ vs. amplitude of oscillation at fixed temperature and end load

2.4 Temperature Compensation

The data reduction can be simplified somewhat if the pendulum is designed to be temperature compensated; that is, if the pendulum gives results which are independent of the temperature. Temperature compensation is achieved by insuring that neither the torsional properties of the suspension wire nor the moment of inertia of the holder change with temperature.

For a single-wire suspension,

$$k = \frac{\pi d^4 G}{32\ell} \quad . \tag{3}$$

Or, $\log k = \text{const.} + 4 \log d + \log G - \log \ell$.

Taking partial derivatives with respect to τ, and noting that

$$\beta = \frac{1}{d}\frac{\partial d}{\partial \tau} = \frac{1}{\ell}\frac{\partial \ell}{\partial \tau} \ ,$$

$$\frac{1}{k}\frac{\partial k}{\partial \tau} = \frac{1}{G}\frac{\partial G}{\partial \tau} + 3\beta. \tag{13}$$

The condition for torsional properties to be independent of temperature is

$$\frac{1}{k}\frac{\partial k}{\partial \tau} = 0.$$

Or

$$3\beta + \frac{1}{G}\frac{\partial G}{\partial \tau} = 0. \tag{14}$$

Ni-Span C is a material whose temperature coefficient of the modulus of rigidity is of the same order of magnitude as its coefficient of thermal expansion. By suitable heat treatment of Ni-Span C, the temperature coefficient of the modulus of rigidity can be made to satisfy Equation (14).

The moment of inertia of the holder will be very nearly constant under temperature variations if it is made from a material with an extremely low coefficient of thermal expansion. For example, if Invar,

a material with a coefficient of expansion equal to $0.9 \times 10^{-6}/^{\circ}C$, were used, a temperature change of $10^{\circ}C$ would produce only a .002 percent change in the moment of inertia of the holder.

Thus, if the suspension wire and the holder meet the above specifications, the torsional pendulum will give results which are virtually independent of the temperature. It should be noted, however, that the moment of inertia of the object being measured is a function of temperature. Thus, if our equipment is sufficiently sensitive we will detect differences in the moment of inertia of a body measured at different temperatures. These will be actual changes in the moment of inertia of the body rather than errors resulting from changes in the instrumentation.

3. EXAMPLES OF SINGLE-WIRE TORSIONAL PENDULUMS

3.1 The Tray Torsional Pendulum

The tray torsional pendulum is designed to measure the moments of inertia of bodies with irregular shapes. Essentially, the apparatus consists of a tray, which holds the object to be measured, suspended by a wire from a rigid supporting arm (Figure 5). The apparatus performs two basic functions: it locates the center of gravity of an object, and it measures I, the moment of inertia of the object about an axis passing through its center of gravity. If desired, the distance R from the axis passing through the center of gravity to any parallel axis may be measured and the moment of inertia about the latter axis, $I_{parallel}$ may be computed by the parallel axis theorem.

$$I_{parallel} = I + mR^2, \tag{15}$$

where m is the mass of the object, $lb\text{-}sec^2/ft$.

3.1.1 Procedure for Locating the Center of Gravity of the Object.

Moveable weights attached to the underside of the tray (Figure 6) permit the operator to shift the center of gravity of the empty tray horizontally until the tray hangs level. The levelness of the tray is determined from a pair of precision levels fixed at right angles on the tray

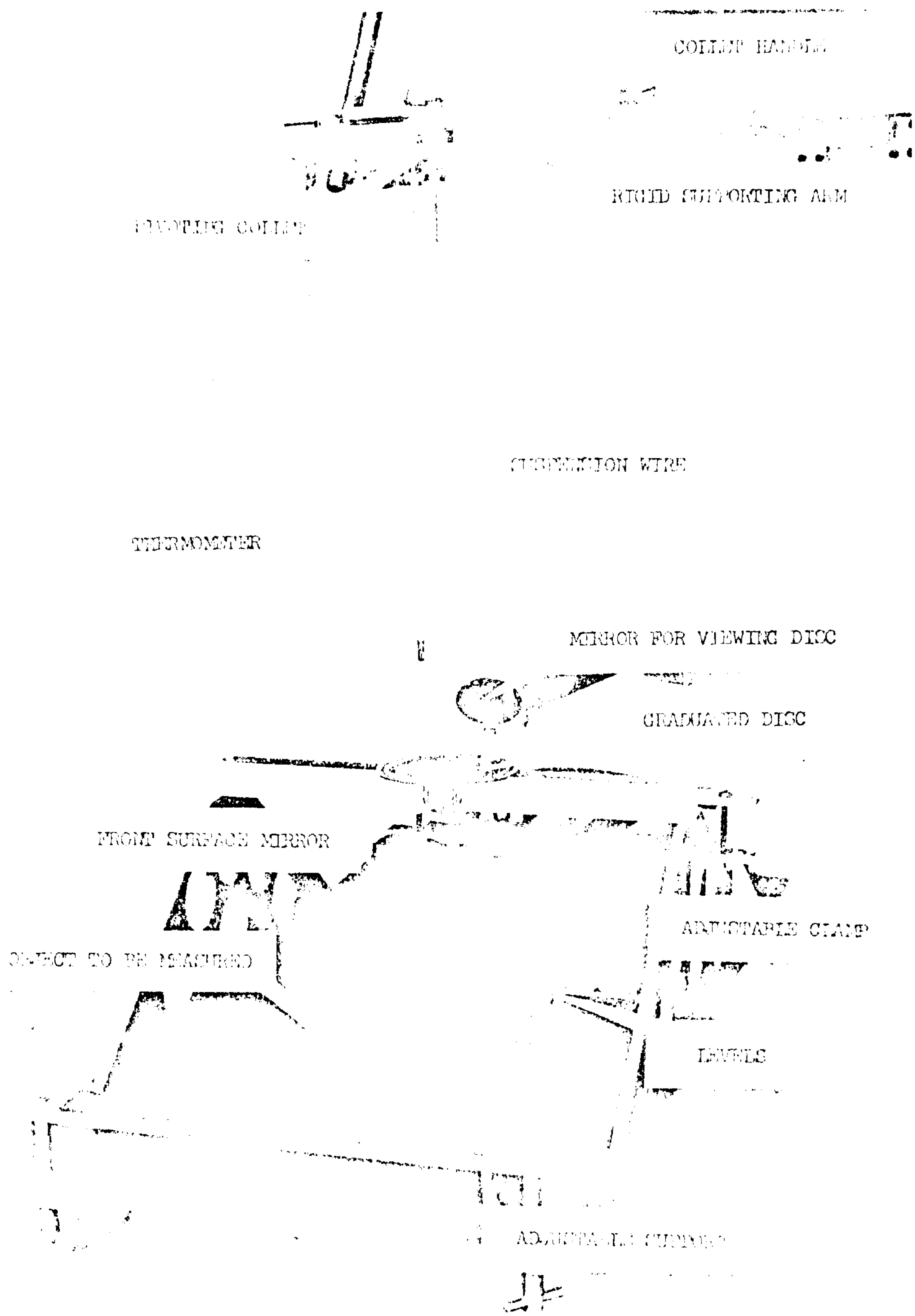

Figure 5. The tray torsional pendulum

(Figure 5). When the tray is level, the suspension wire is vertical, and the center of gravity of the tray is colinear with the suspension wire. After the tray has been leveled the balancing weights are not touched.

The tray is equipped with a clamp at one end and a support at the other end, both of which can be adjusted without shifting the center of gravity of the tray horizontally and affecting the level of the tray. These adjustable fixtures permit the operator to fix, within limits, the attitude of the object to be measured relative to the tray so that the axis about which the moment of inertia is desired will be colinear with the suspension wire. After the empty tray is leveled, the object to be measured is placed in the tray and shifted horizontally until the levels again read level. Then the center of gravity of the object, the center of gravity of the tray, and the vertical suspension wire are all colinear.

A conical-tipped stylus, inserted through a hole in the bottom of the tray directly beneath the suspension wire and perpendicular to the tray, marks the location of the center of gravity on the object. Figure 6 illustrates the marking procedure. This marks the intersection of the vertical axis passing through the center of gravity of the object and the surface of the object.

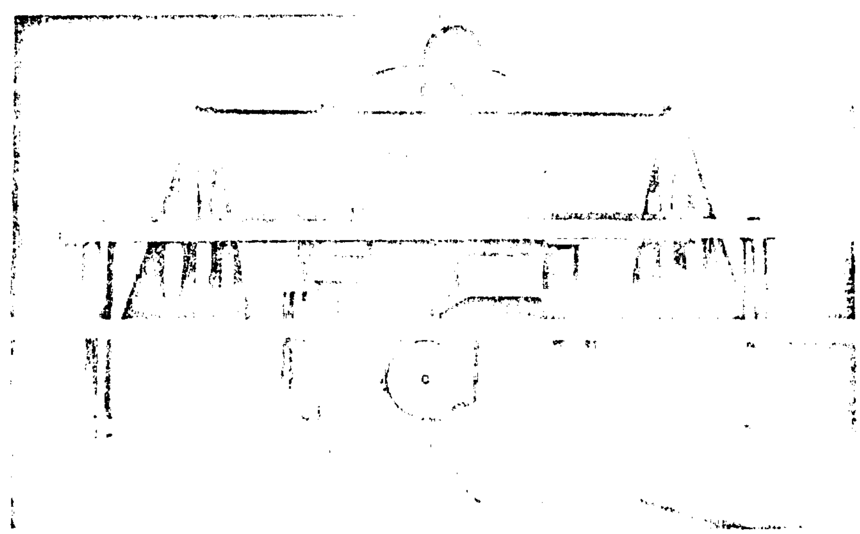

Figure 6. Marking the center of gravity of an object

3.1.2 Procedure for Measuring the Moment of Inertia of an Odd-Shaped Body. The body to be measured is placed in the tray, aligned and balanced as described in 3.1.1. The pendulum is set into torsional oscillation about the suspension wire by rotating the collet handle (Figure 5) through a small angle ($\approx 10^{\circ}$) and returning it to its original position. This permits the tray to oscillate about the vertical suspension without inducing pitching or side-to-side motion in the tray. The period of oscillation is then measured by the photo-electric circuitry described in Section 4. The above procedure is used for both the calibration masses and the object whose moment of inertia is desired.

The values of the periods, T_i, and the known moments of inertia, I_i, of the calibration masses, together with the observed values of temperature, weight, and amplitude of oscillation for each calibration run, are substituted into Equation (11). Temperature is monitored by reading a thermometer suspended near the apparatus. The amplitude of oscillation is determined by using a transit and reading the angular deflection of the graduated disc (Figure 5) mounted coaxially with the suspension wire. Equation (11) is solved as described in Section 2 for k_o and I_{ho}. The computed values of k_o and I_{ho}, together with the measured values of period, temperature, weight and amplitude of oscillation for the odd-shaped object, are substituted into Equation (12), which permits the calculation of the unknown moment of inertia.

When high accuracy is not required, k and I_h may be assumed constant and Equations (6) and (7) will be used for the calculation of the moment of inertia. An alternate method when high accuracy is not required is a graphical one. The known moments of inertia of the calibration masses are plotted against the squares of their respective periods and fitted with a straight line. Then, the period of oscillation with an object in the tray is determined, and the moment of inertia of the object is read from the graph.

3.2 A Torsional Pendulum for Measuring Moments of Inertia of Projectiles

Generally speaking, projectiles have two distinct principal moments of inertia: the moment of inertia about the longitudinal axis (axial moment of inertia), and the moment of inertia about an axis passing through the center of gravity and perpendicular to the longitudinal axis (transverse moment of inertia). Moments of inertia and products of inertia about other axes can be computed from the principal moments of inertia and the inclination of the axes to the principal axes.

Described below is an apparatus* which can be used to measure the axial and transverse moments of inertia of projectiles varying in caliber from 37 mm to 105 mm, including projectiles with sabots.

The apparatus can also be used to measure the distance from the base of a projectile to its center of gravity. The quantities measured and the calculations are the same as described in Sections 2 and 3.

The apparatus consists of holding devices in which the projectile to be measured is mounted, a single-wire suspension, and a rigid supporting arm. Figure 7 gives an overall view of this torsional pendulum.

Two types of holding devices are used:

(1) a yoke to hold the projectile with its transverse axis colinear with the suspension wire (Figure 7)

(2) an axial holder to hold the projectile with its longitudinal axis colinear with the suspension wire (Figure 8).

Both holders screw into a common mounting fixture which is clamped to the bottom of the suspension wire. This maintains a constant length of suspension and eliminates the necessity of recalibrating the pendulum when the holders are interchanged. As shown in Figure 7, the mounting fixture includes a front surface mirror which is used in the photoelectric timing circuit described in Section 4. A graduated disc on

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* *The torsional pendulum described here is generally similar to that described in Reference 3, but is somewhat more versatile.*

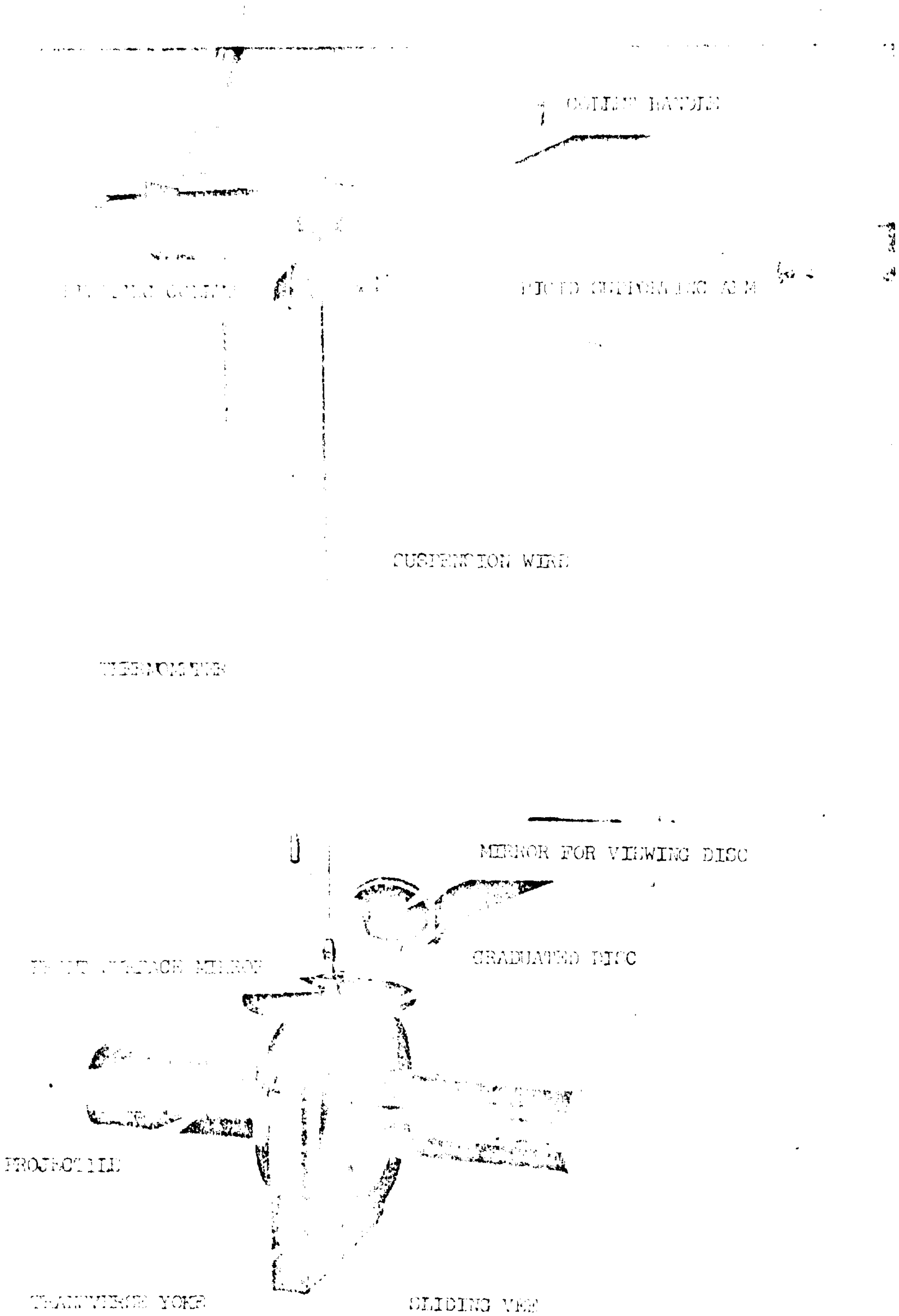


Figure 7. Torsional pendulum for measuring moments of inertia of projectiles

the mounting fixture is used in monitoring the amplitude of oscillation. The mounting fixture is balanced about its point of attachment to the suspension wire so that its center of gravity is colinear with the suspension. Likewise, the axial and transverse holders are balanced about their point of attachment to the mounting fixture so that their centers of gravity are colinear with the suspension.

3.2.1 Procedure for Measuring the Axial Moment of Inertia. The axial holder clamps to a projectile or calibration mass by means of three screws threaded into the side of the holder 120° apart. The holder is usually designed to fit snugly over one end of a projectile so that the axis of the projectile and the axis of the holder are aligned. If more than a few mils clearance exists between the holder and the projectile, they are aligned by means of a Vee block and a dial gage as illustrated in Figure 8. As the projectile is rotated in the Vee block, the clamping screws are adjusted to give minimum deflection of the dial gage.

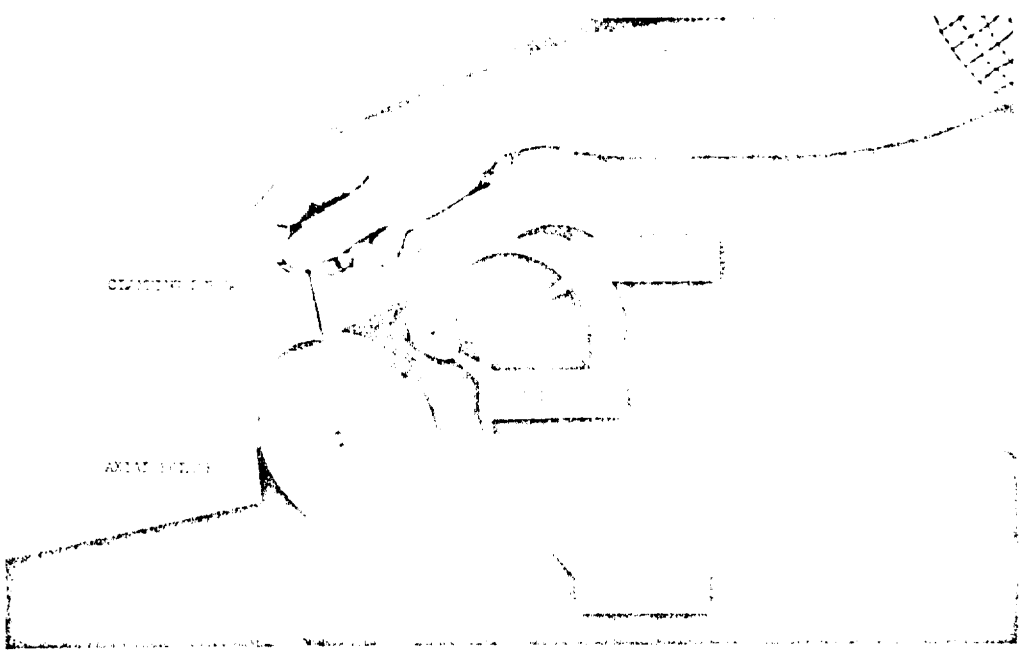


Figure 8. Alignment of the axial holder with the longitudinal axis of a projectile

If there is considerable static unbalance[3,4] present, and the magnitude and orientation of the unbalance is known, the Vee block-dial gage technique can be used to align the center of gravity of the projectile with the axis of the holder. As the projectile is rotated in the Vee block, the clamping screws are adjusted to give a maximum deflection of the dial gage in the direction of the unbalance. For proper alignment, the maximum deflection less the minimum deflection of the dial gage should be equal to twice the magnitude of the unbalance (defined as the distance between the longitudinal axis of a projectile and its center of gravity). Figure 9 illustrates the relationships involved.

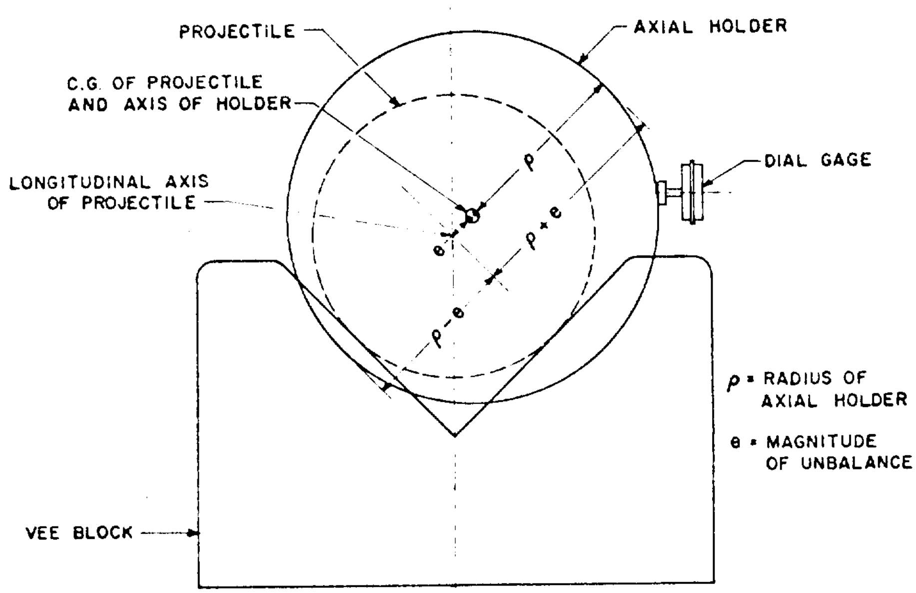


Figure 9. Alignment of the center of gravity of an unbalanced projectile with the axial holder

After the projectile is aligned with the holder as outlined above, the projectile-holder combination is screwed to the mounting fixture. The threads on the mounting fixture are coaxial with the suspension wire while the threads on the holder are coaxial with the holder. Thus, the

longitudinal axis of the projectile will be aligned with the suspension wire. Or, in the case of a statically unbalanced projectile, the axis passing through the center of gravity of the projectile and parallel to its longitudinal axis will be aligned with the suspension wire.

3.2.2 Procedure for Measuring the Transverse Moment of Inertia. The transverse yoke is balanced about its point of attachment to the mounting fixture. When the yoke is screwed into the mounting fixture, both the suspension wire and the yoke hang vertically. When a projectile is placed in the yoke it will tip unless the center of gravity of the projectile is colinear with the suspension wire. The supporting Vees of the yoke are designed with a plane of symmetry which contains the suspension wire. When a projectile with its center of gravity located on the longitudinal axis is placed in the yoke, its center of gravity will automatically lie in the given plane of symmetry. Thus, it is only necessary to shift the center of gravity of the projectile within the plane of symmetry until it coincides with the suspension wire. The positioning of the center of gravity of the projectile is accomplished by means of the horseshoe-like tool shown in Figure 10.

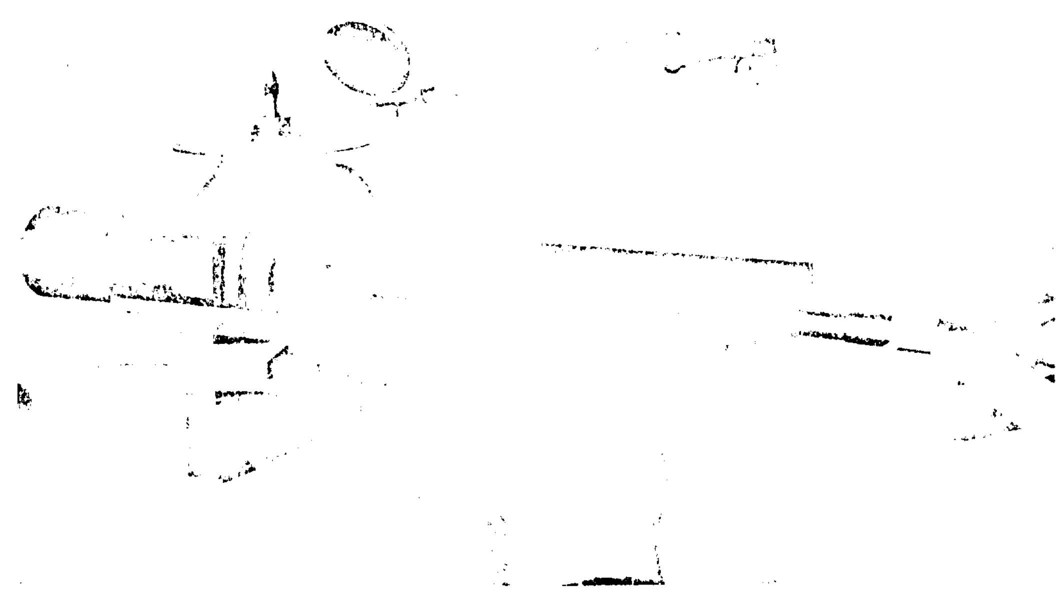


Figure 10. Positioning the projectile in the transverse yoke

With this tool fitted over the face of the yoke, the turning action of the micrometer screw pushes the projectile through the yoke in the direction shown in Figure 10. The projectile is positioned until the yoke and the suspension wire hang vertically. The perpendicularity of the yoke and the suspension wire can be judged by aligning them with the vertical cross hair of a transit.

The positioning tool is also used to measure the distance from the base of a projectile to its center of gravity. When the yoke and the suspension wire hang vertically, the center of gravity of the projectile is equidistant from the faces of the yoke. The distance from the center of gravity to the base of the micrometer, which is a constant determined by the geometry of the tool and the yoke, less the micrometer reading when the yoke hangs vertically, is equal to the distance from the base of the projectile to its center of gravity.

A sliding Vee (Figures 7 and 10) on one face of the yoke permits the accommodation of projectiles whose centers of gravity are not located in cylindrical portions. (Take, for example, a saboted projectile whose center of gravity lies on the longitudinal axis at one edge of the sabot.) The sliding Vee slides in a direction parallel to the suspension wire. Hence the center of gravity of the yoke remains colinear with the suspension wire, and the moment of inertia of the yoke about the suspension remains constant. The sliding Vee is fixed in a position so that the longitudinal axis of the projectile is perpendicular to the faces of the yoke. Then, if the yoke and the suspension wire hang vertically, the centerline of the suspension wire will coincide with the transverse axis of the projectile.

4. NOTES ON MEASUREMENT TECHNIQUES

Moment of inertia is an indirectly measured quantity. That is, it is computed from several directly measured quantities, the most important being the period of torsional oscillation. The accuracy of the measured value of the moment of inertia depends primarily upon the accuracy with

which the period of oscillation is measured. Modern crystal-controlled electronic timers have the capability of measuring short periods of time with an accuracy of one-millionth of a second. However, no matter how good a timer is, unless it is started and stopped at corresponding (that is, truly periodic) points of the torsional cycle, erroneous readings can result. The procedure outlined here should minimize errors in the measurement.

Following common practice in measuring frequency of vibrations, a photoelectric triggering circuit is used, whereby a narrow collimated beam of light is reflected from a small mirror mounted on the pendulum onto a photocell. A schematic of the apparatus is shown in Figure 11.

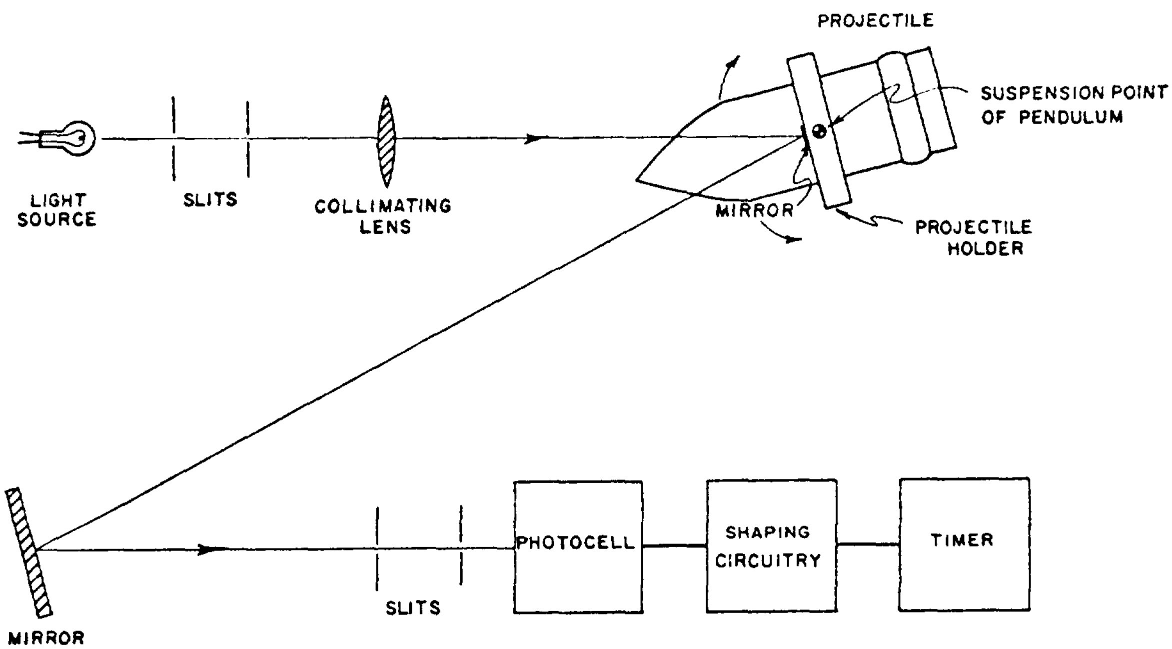


Figure 11. Schematic plan of experimental apparatus

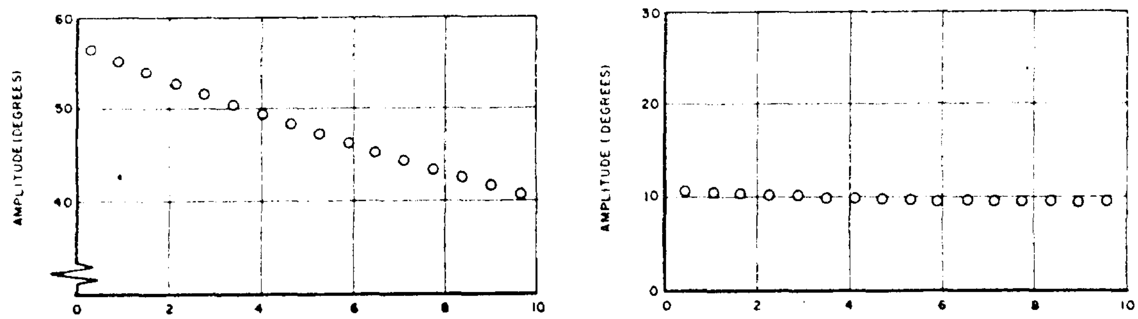
The reflected beam of light traverses the photocell twice during each cycle of the torsional oscillation. The photocell, in turn, starts and stops the timer, which measures the period of oscillation.

Generally, the higher the traversing speed of the light beam across the photocell, the faster the rise time of the photocell signal and the more accurate the timing of the period. Thus, it is desirable to maximize the traversing speed of the light beam. For small angles, the angular velocity of the reflected light beam is twice the angular velocity of the pendulum. Therefore, the speed with which the light beam traverses the photocell is given by

$$V = 2L\dot{\theta} \quad , \tag{17}$$

where L is the distance the light travels from the pendulum to the photocell. L can be increased by the use of an additional mirror to fold the light beam, as shown in Figure 11. The angular velocity, $\dot{\theta}$, of the pendulum is a maximum when the pendulum passes through its rest position; i.e., when $\theta = 0$. Thus for maximum traversing speed with a given L the optical system should be aligned so that the light beam falls on the photocell when the pendulum is at rest.

As noted in Section 2, the torsional oscillations are subjected to viscous damping due to the surrounding air. As the pendulum oscillates the amplitude decreases by an amount proportional to the amplitude at any moment (see Figure 12a).



Time (minutes)

(a)

Time (minutes)

(b)

Figure 12. Amplitude vs. time for (a) large amplitudes and (b) small amplitudes

Further, the period of oscillation changes with amplitude resulting in considerable change in the period as well as the amplitude as damping progresses. This situation leads to uncertainty as to what period T corresponds to what amplitude Θ for large amplitudes. Thus it is advantageous to choose a small amplitude of oscillation such that the amplitude envelope is almost flat as shown in Figure 12b. In this region the amplitude and period change negligibly, which enables one to time several cycles, and find an average period $\overline{T}$ corresponding to the average amplitude during recording. This procedure greatly reduces random errors in the measured value of the period resulting from simple pendular or pitching oscillations superimposed on the torsional oscillation. Of course, bringing the pendulum to a state of rest before setting it into torsional oscillation helps to minimize these extraneous oscillations. However, due to disturbing factors like air currents, ground vibration and imperfect dynamic balance of the pendulum, these secondary modes of oscillation will be excited to some extent. Air currents can be largely eliminated by sealing off sources of drafts and shutting off fans, air conditioning, and similar equipment. Obviously, the pendulum should not be located in an area subject to heavy vibration or shock.

If dynamic imbalance exists with respect to the axis of rotation (and it will exist in the general case of non-symmetric distribution of mass about an axis) it will give rise to torques and oscillations about axes perpendicular to the initial axis of rotation. These torques are proportional to the angular velocity and acceleration of the pendulum about the axis of rotation. These oscillations can be kept small by keeping the initial amplitude small. The maximum amplitude that can be used without inducing significant secondary oscillations should be determined experimentally for each body-holder combination. This can be done by aligning the lower portion of the suspension wire with the vertical cross hair of a transit, and noting whether there is any displacement of the wire relative to the cross hair when the pendulum is oscillating with a given amplitude.

The amplitude can be monitored by using a transit to view a protractor dial mounted on the pendulum coaxially with the line of suspension. The vertical cross hair is centered on the 0° mark of the protractor when the pendulum is at rest. When the pendulum is set into oscillation, the amplitude is equal to the largest angle of the protractor traversing the cross hair of the telescope.

The temperature can be monitored simultaneously with the amplitude by placing a thermometer in the same field of view of the telescope as the protractor dial, as shown in Figure 5. The advantage in using a telescope to monitor temperature and amplitude is that it removes the operator from the vicinity of the pendulum, and thus prevents human interference with the measurements.

ACKNOWLEDGEMENTS

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REFERENCES

1. L. Campbell and G. Beck, "The Instruction Code for the BRL Electronic Scientific Computer (BRLESC)," Ballistic Research Laboratories Report No. 1379, November 1961.

2. L. Campbell and G. Beck, "The FORAST Programming Language for ORDVAC and BRLESC (Revised)," Ballistic Research Laboratories Report No. 1283, March 1965.

3. E. Dickinson, "Physical Measurements of Projectiles," Ballistic Research Laboratories Technical Note No. 874, February 1954.

4. R. Kirkendall, "Apparatus for Measuring the Static Unbalance of Projectiles," Ballistic Research Laboratories Memorandum Report No. 1564, May 1964.

APPENDIX A

THE EFFECT OF DAMPING ON MOMENT OF INERTIA MEASUREMENTS

If a viscous damping term is added to Equation (1), we obtain

$$I_b \ddot{\theta} + C\dot{\theta} + k\theta = 0, \qquad \text{(A-1)}$$

which has the well known solution,

$$\theta = e^{-\frac{C}{2I_b} t} (\Theta_1 \cos \omega t + \Theta_2 \sin \omega t), \qquad \text{(A-2)}$$

where

$$\omega = \frac{2\pi}{T} = \sqrt{\frac{k}{I_b} - \frac{C^2}{4I_b{}^2}} \quad . \qquad \text{(A-3)}$$

Solving for I_b:

$$I_b = \frac{kT^2}{8\pi} + \sqrt{\frac{k^2T^4}{64\pi^4} - \frac{C^2T^2}{16\pi^2}}$$

$$\approx \frac{kT^2}{4\pi^2} - \frac{C^2}{4k} \quad . \qquad \text{(A-4)}$$

Using Equations (2) and (A-4) we obtain a more exact expression for I:

$$I = \frac{kT^2}{4\pi^2} - \frac{C^2}{4k} - I_h \quad . \qquad \text{(A-5)}$$

This is the same as Equation (7) except for the small term $\frac{C^2}{4k}$ arising from viscous damping. The fractional error in I rising from omission of this damping term is

$$\frac{\Delta I}{I} = \frac{C^2}{4kI} \quad .$$

For most torsional pendulums, this turns out to be a negligible error. For the tray torsional pendulum described in Section 3.1, typical values are C = .0004 ft-lb-sec, k = .002 ft-lb, and I = .15 ft-lb-sec^2. In this case, $\Delta I/I = \frac{(.0004)^2}{4(.002)(.15)}$ = .013 percent. This error is an order

of magnitude smaller than the error in I arising from other quantities, such as the calculated moments of inertia of the calibration masses. Therefore, in this case the inclusion of a damping term in the calculations for I is not necessary. For other pendulums with higher damping coefficients and smaller restoring torques, inclusion of a damping term may be justified.

APPENDIX B

AUTOMATED DATA REDUCTION PROGRAM FOR
MOMENT OF INERTIA MEASUREMENTS

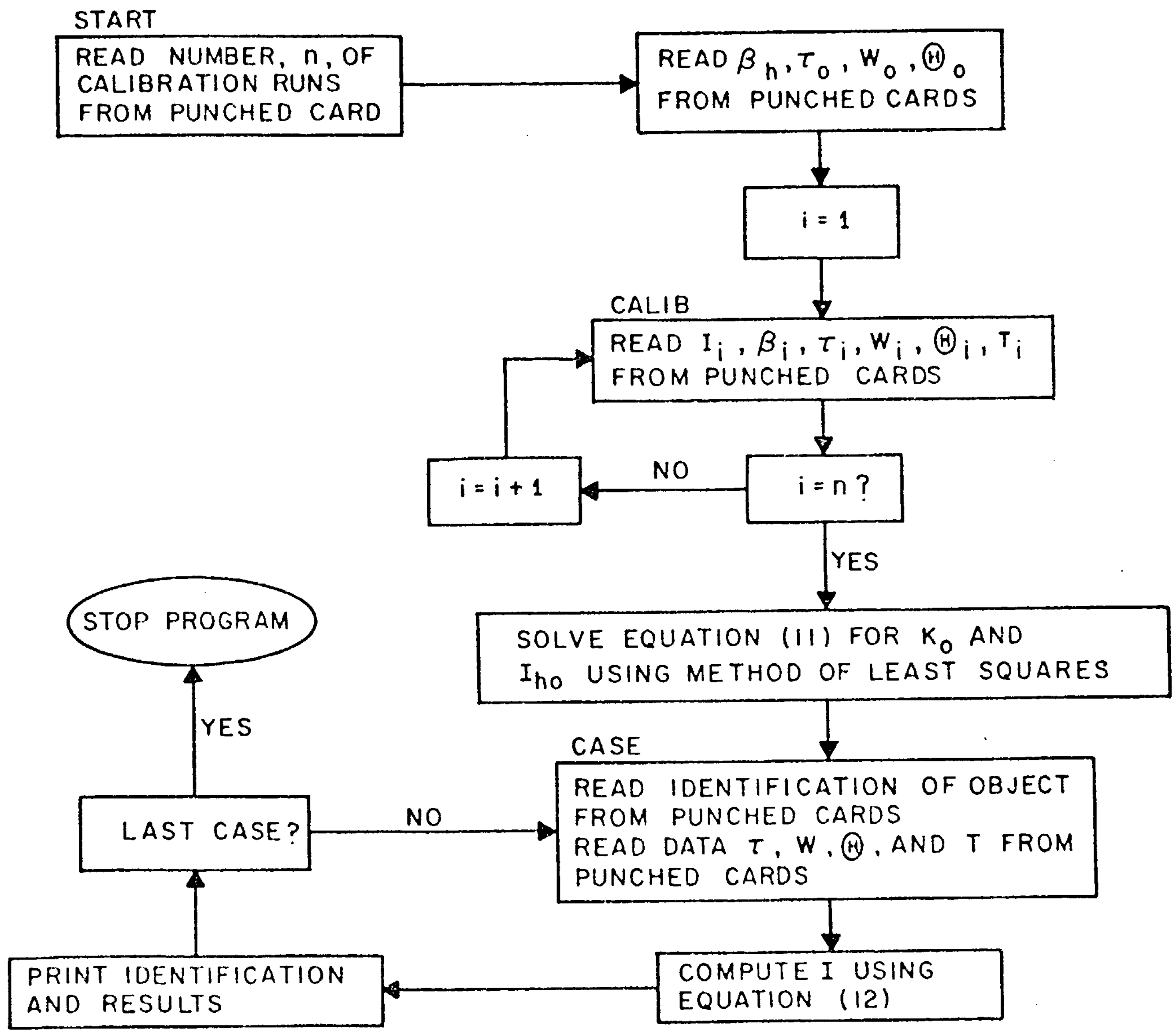
1. Simplified Flow Chart

2. Typical Input Data

3. Typical Results

AUTOMATED DATA REDUCTION PROGRAM FOR MOMENT OF INERTIA MEASUREMENTS

SIMPLIFIED FLOW CHART



AUTOMATED DATA REDUCTION PROGRAM FOR MOMENT OF INERTIA MEASUREMENTS

TYPICAL INPUT DATA

I. Constants

Number of Calibration Runs, n = 3

β_h = .000025 per $^{\circ}C$

τ_o = 21.0 $^{\circ}C$

W_o = [illegible] lb

θ_o = 10.0 degrees

II. Calibration Data

Run, i	I_i(ft-lb-sec^2)	β_i(per $^{\circ}C$)	τ_i($^{\circ}C$)	W_i(lb)	θ_i(deg)	T_i(sec)
1	.04475	.000019	23.0	4.965	9.0	14.2508
2	.14135	.000019	23.0	9.935	14.5	15.3491
3	.24621	.000019	23.0	14.800	24.0	17.2905

III. Measurement Data

Identification	τ($^{\circ}C$)	W(lb)	θ(deg)	T(sec)
SPR, unloaded, horizontal axis through C.G. ⊥ bore	23.0	7.43	17.5	15.7248
SPR, 60 rounds, horizontal axis through C.G. ⊥ bore	23.0	8.37	15.0	15.7692
SPR, unloaded, launcher, horizontal axis through C.G. ⊥ bore	23.0	11.55	13.5	16.4564
SPR, 60 rounds, launcher, 3 grenades, horizontal axis through C.G. ⊥ bore	23.6	13.38	13.0	16.7614
AAIC, unloaded, horizontal axis through C.G. ⊥ bore	24.5	6.59	20.0	15.7426
AAIC, 60 rounds, horizontal axis through C.G. ⊥ bore	24.5	7.41	25.0	15.8036
AAIC, unloaded, launcher, horizontal axis through C.G. ⊥ bore	23.3	9.41	29.0	16.8816
AAIC, 60 rounds, launcher, 3 grenades, horizontal axis through C.G. ⊥ bore	23.3	12.59	15.5	17.1550

AUTOMATED DATA REDUCTION PROGRAM FOR MOMENT OF INERTIA MEASUREMENTS

TYPICAL RESULTS

```
AUG.30,67  BRLESC  FORAST F62
     PROB 1664AM-MI   ROBERT W. DEAS    3203  MOMENTS OF INERTIA


SPR RIFLE, UNLOADED, HORIZ AXIS THRU C.G. AND NORMAL TO BORE
MOMENT OF INERTIA=  .14061 FT-LB-SEC**2

SPR RIFLE, 60 RDS, HORIZ AXIS THRU C.G. AND NORMAL TO BORE
MOMENT OF INERTIA=  .14335 FT-LB-SEC**2

SPR RIFLE, UNLOADED, LAUNCHER, HORIZ AXIS THRU C.G. AND NORMAL TO BORE
MOMENT OF INERTIA=  .18936 FT-LB-SEC**2

SPR RIFLE, 60 RDS, LAUNCHER, 3 GRENADES, HORIZ AXIS THRU C.G. AND NORMAL TO BORE
MOMENT OF INERTIA=  .21005 FT-LB-SEC**2

AAIC RIFLE, UNLOADED, HORIZ AXIS THRU C.G. AND NORMAL TO BORE
MOMENT OF INERTIA=  .14211 FT-LB-SEC**2

AAIC RIFLE, 60 RDS, HORIZ AXIS THRU C.G. AND NORMAL TO BORE
MOMENT OF INERTIA=  .14480 FT-LB-SEC**2

AAIC RIFLE, UNLOADED, LAUNCHER, HORIZ AXIS THRU C.G. AND NORMAL TO BORE
MOMENT OF INERTIA=  .21746 FT-LB-SEC**2

AAIC RIFLE, 60 RDS, LAUNCHER, 3 GRENADES, HORIZ AXIS THRU CG AND NORMAL TO BORE
MOMENT OF INERTIA=  .23573 FT-LB-SEC**2
```

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13. ABSTRACT

The theory of single-wire torsional pendulums, including second order effects, is discussed. The theory is applied to the use of torsional pendulums in measuring moments of inertia. Two types of torsional pendulums designed for moment of inertia measurements are described: one for odd-shaped bodies and one for artillery projectiles. A data reduction program for digital computers is included.

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Data Reduction						
Artillery Projectiles						

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